D1-82-0261

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Mathematics Research

May 1963

D1-82-0261

LAMINAR FREE CONVECTION FROM A NON-ISOTHERMAL HORIZONTAL CYLINDER

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Mathematical Note No. 303

Mathematics Research Laboratory

BOEING SCIENTIFIC RESEARCH LABORATORIES

May 1963

SUMMARY

Laminar free convection from a non-isothermal horizontal cylinder is analysed. The wall surface temperature is assumed to be varied in the manner of $a_1(\frac{x}{R})^2 + a_2(\frac{x}{R})^4$. Special transformations are devised and employed so that the resulting differential equations and boundary conditions are free of the parameters a_1 and a_2. These differential equations aré solved once and for all; solutions to the original equations for any particular values of a_1 and a_2 may then be read off easily as linear combinations of the numerical solutions given here. It is found that the dependence of heat transfer from a horizontal cylinder on Prandtl number is practically the same as that from a vertical plate. Furthermore, the heat transfer is greatly influenced by the surface temperature variations.

INTRODUCTION

Laminar free convective heat transfer from a cylinder has been studied theoretically by Hermann [1] and by Chiang and Kaye [2]. Both references report the solutions of the pertinent boundary layer equations for the case of an isothermal wall surface, the first for a Prandtl number of 0.74 and the second for a Prandtl number of 0.7. The closely related problem of the laminar free convective heat transfer from a vertical plate has been analysed by Ostrach [3] for an isothermal wall and with a wide range of Prandtl numbers. When either the plate surface is non-isothermal or the surface heat flux is prescribed, the free convection problem has been solved approximately by Sparrow [4]. Exact formulations and solutions have been reported by Sparrow and Gregg in Refs. [5] and [6] for uniform heat flux and non-isothermal surface respectively. A search of the literature reveals that there is no solution available for the prediction of free convective heat transfer to or from a cylinder when the surface is non-isothermal. As demonstrated in [6], the heat transfer from a non-isothermal plate is markedly different from that of an isothermal plate. It is reasonable to suppose that the free convective heat transfer from a non-isothermal cylinder would be also significantly different from that of an isothermal one. Since in a great many technical applications, the cylindrical surface from which heat is being transferred is non-isothermal, it is the purpose of this paper to report the free convective heat transfer from a non-isothermal cylinder. The surface temperature is assumed to

take the following form:

$$(T_w - T_\infty) = (T_{wo} - T_\infty)[1 + a_1(\frac{x}{R})^2 + a_2(\frac{x}{R})^4] \qquad (1)$$

where a_1 and a_2 are arbitrary constants. The surface temperature may be either higher or lower than the free stream temperature. If the surface temperature is higher than the free stream temperature, x must be measured from the lower stagnation point (Figure 1a). On the other hand, if the surface temperature is lower than the free stream temperature, x must be measured from the upper stagnation point (Figure 1b). When the coordinate systems are taken in this manner, the method of analysis and the results for the heat transfer parameters are the same for either $T_w > T_\infty$ or $T_w < T_\infty$, and there will be no need to treat them separately. The analysis will be carried out for the case of $T_w > T_\infty$, but it is to be remembered that the results apply to both cases depicted in Figure 1.

GOVERNING DIFFERENTIAL EQUATIONS

The equations expressing conservation of mass, momentum, and energy for steady laminar flow in a boundary layer on a horizontal cylinder are as follows*:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)\sin\frac{x}{R} \qquad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\frac{\partial^2 T}{\partial y^2}. \qquad (4)$$

---

* See page 17 for a listing of the definitions of the various variables involved.

The density has been considered a variable only in forming the buoyancy force $g\beta(T - T_\infty)\sin\frac{x}{R}$. Variations of all other properties are neglected. Viscous dissipation and work against the gravity field also have been omitted from the energy equation (4) since they are negligibly small.

The appropriate boundary conditions are:

$$\begin{aligned} &y = 0 \qquad u = v = 0 \qquad T = T_w(x) \\ &y \to \infty \qquad u \to 0 \qquad T \to T_\infty \end{aligned} \tag{5}$$

Equation (2) can be automatically satisfied if a stream function is introduced such that

$$u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial x} \tag{6}$$

The momentum and energy equations may be rewritten in the following forms:

$$M_\eta M_{\bar{x}\eta} - M_{\eta\eta} M_{\bar{x}} = M_{\eta\eta\eta} + \phi \sin\bar{x} \tag{7}$$

$$M_\eta \phi_{\bar{x}} - \phi_\eta M_{\bar{x}} = \frac{1}{Pr}\phi_{\eta\eta} \tag{8}$$

where

$$\begin{aligned} \bar{x} &= \frac{x}{R} \\ \eta &= \left[\frac{g\beta(T_{wo} - T_\infty)R^3}{\nu^2}\right]^{\frac{1}{4}} \frac{y}{R} = (Gr)^{\frac{1}{4}}\frac{y}{R} \\ M &= \left[\frac{\nu^2}{g\beta(T_{wo} - T_\infty)R^3}\right]^{\frac{1}{4}} \frac{\psi}{\nu} = \frac{1}{(Gr)^{\frac{1}{4}}}\frac{\psi}{\nu} \\ \phi &= \frac{T - T_\infty}{T_{wo} - T_\infty} \end{aligned} \tag{9}$$

In terms of the new variables, the boundary conditions are

$$\begin{aligned} &\eta = 0 \qquad M = M_{\bar{x}} = M_\eta = 0, \qquad \phi = 1 + a_1\bar{x}^2 + a_2\bar{x}^4 \\ &\eta \to \infty \qquad M_\eta \to 0 \qquad \phi \to 0 \end{aligned} \tag{10}$$

Similar to [1], equations (7) and (8) together with the boundary conditions (10) are solved by a perturbation technique. It is assumed that M and ϕ take the following forms:

$$\begin{aligned} M &= \bar{x}F_o(\eta) + \bar{x}^3F_1(\eta) + \bar{x}^5F_2(\eta) + \cdots \\ \phi &= G_o(\eta) + \bar{x}^2G_1(\eta) + \bar{x}^4G_2(\eta) + \cdots \end{aligned} \tag{11}$$

When equations (11) are substituted into equations (7) and (8), and equal powers of $\bar{x}$ are collected, the following set of ordinary differential equations is obtained:

$$\begin{aligned} &F_o''' + F_oF_o'' - F_o'^2 + G_o = 0 \\ &\frac{1}{Pr} G_o'' + F_oG_o' = 0 \end{aligned} \tag{12}$$

$$\begin{aligned} &F_1''' + F_oF_1'' - 4F_o'F_1' + 3F_o''F_1 + G_1 - \frac{1}{6} G_o = 0 \\ &\frac{1}{Pr} G_1'' + F_oG_1' - 2F_o'G_1 + 3G_o'F_1 = 0 \end{aligned} \tag{13}$$

$$\begin{aligned} &F_2''' + F_oF_2'' - 6F_o'F_2' + 5F_o'' F_2 + 3F_1F_1'' - 3F_1'^2 \\ &\qquad + G_2 - \frac{1}{6}G_1 + \frac{1}{120} G_o = 0 \\ &\frac{1}{Pr} G_2'' + F_oG_2' - 4F_o'G_2 + 3F_1G_1' - 2F_1'G_1 + 5F_2G_o' = 0 \end{aligned} \tag{14}$$

where the superscript prime (') denotes differentiation with respect to η.

Notice that in obtaining equations (12) and (14), $\sin \bar{x}$ in equation (7) has been approximated by the first three terms of its power series. For $\bar{x} = \frac{\pi}{2}$ and $\frac{2\pi}{3}$, the errors resulting from this truncation are about 0.5% and 4% respectively.

The boundary conditions are

$$
\begin{array}{lll}
\eta = 0 & F_0 = F' = 0 & G_0 = 1 \\
 & F_1 = F_1' = 0 & G_1 = a_1 \\
 & F_2 = F_2' = 0 & G_2 = a_2 \\
\eta \to \infty & F_0' \to 0 & G_0 \to 0 \\
 & F_1' \to 0 & G_1 \to 0 \\
 & F_2' \to 0 & G_2 \to 0
\end{array}
\tag{15}
$$

An inspection of equations (12) to (15) reveals that the present problem involves three parameters, Pr, a_1, and a_2. For each combination of Pr, a_1, and a_2, it would be necessary to solve three pairs of simultaneous equations. If the fluid is a gas, the Prandtl number is relatively constant. However, both a_1 and a_2 can have a wide range of values, and hence it might be necessary to solve equations (12) to (15) for a great many combinations of a_1 and a_2. This is obviously impractical,if not impossible. Consequently, the following transformations (Eq. 16) are used to transform the original equations and boundary conditions to a new set of differential equations and boundary conditions

which are free of the parameters a_1 and a_2.

$$
\begin{aligned}
F_o &= X_1 & G_o &= Y_1 \\
F_1 &= a_1X_2 + X_3 & G_1 &= a_1Y_2 + Y_3 \\
F_2 &= a_2X_4 + a_1^2X_5 + a_1X_6 + X_7 & G_2 &= a_2Y_4 + a_1^2Y_5 + a_1Y_6 + Y_7
\end{aligned}
\tag{16}
$$

The new differential equations together with their boundary conditions are

$$
\left.
\begin{aligned}
& X_1''' + X_1X_1'' = X_1'^2 - Y_1 \\
& Y_1'' + \Pr X_1Y_1' = 0 \\
& X_1(0) = X_1'(0) = X_1'(\infty) = 0 \\
& Y_1(0) = 1 \quad Y_1(\infty) = 0
\end{aligned}
\right\} \tag{I}
$$

$$
\left.
\begin{aligned}
& X_2''' + X_1X_2'' = 4X_1'X_2' - 3X_1''X_2 - Y_2 \\
& Y_2'' + \Pr[X_1Y_2' - 2X_1'Y_2 + 3X_2Y_1'] = 0 \\
& X_2(0) = X_2'(0) = X_2'(\infty) = 0 \\
& Y_2(0) = 1 \quad Y_2(\infty) = 0
\end{aligned}
\right\} \tag{II}
$$

$$
\left.
\begin{aligned}
& X_3''' + X_1X_3'' = 4X_1'X_3' - 3X_1''X_3 - Y_3 + \tfrac{1}{6}Y_1 \\
& Y_3'' + \Pr[X_1Y_3' - 2X_1'Y_3 + 3X_3Y_1'] = 0 \\
& X_3(0) = X_3'(0) = X_3'(\infty) = 0 \\
& Y_3(0) = Y_3(\infty) = 0
\end{aligned}
\right\} \tag{III}
$$

$$\begin{aligned}
&X_4''' + X_1X_4'' = 6X_1'X_4' - 5X_1''X_4 - Y_4\\
&Y_4 + Pr[X_1Y_4' - 4X_1'Y_4 + 5X_4Y_1'] = 0\\
&X_4(0) = X_4'(0) = X_4'(\infty) = 0\\
&Y_4(0) = 1 \quad Y_4(\infty) = 0
\end{aligned} \tag{IV}$$

$$\begin{aligned}
&X_5''' + X_1X_5'' = 6X_1'X_5' - 5X_1''X_5 + 3(X_2'^2 - X_2X_2'') - Y_5\\
&Y_5'' + Pr[X_1Y_5' - 4X_1'Y_5 + 3X_2Y_2' - 2X_2'Y_2 + 5X_5Y_1'] = 0\\
&X_5(0) = X_5'(0) = X_5'(\infty) = 0\\
&Y_5(0) = Y_5(\infty) = 0
\end{aligned} \tag{V}$$

$$\begin{aligned}
&X_6''' + X_1X_6'' = 6X_1'X_6' - 5X_1''X_6 + 3(2X_2'X_3' - X_2X_3'' - X_3X_2'') - Y_6 + \frac{Y_2}{6}\\
&Y_6'' + Pr[X_1Y_6' - 4X_1'Y_6 + 3(X_2Y_3' + X_3Y_2') - 2(X_2'Y_3 + Y_2X_3') + 5X_6Y_1'] = 0\\
&X_6(0) = X_6'(0) = X_6'(\infty) = 0\\
&Y_6(0) = Y_6(\infty) = 0
\end{aligned} \tag{VI}$$

$$\begin{aligned}
&X_7''' + X_1X_7'' = 6X_1'X_7' - 5X_1''X_7 + 3(X_3'^2 - X_3X_3'') - Y_7 + \frac{Y_3}{6} - \frac{Y_1}{120}\\
&Y_7'' + Pr[X_1Y_7' - 4X_1'Y_7 + 3X_3Y_3' - 2X_3'Y_3 + 5X_7Y_1' = 0\\
&X_7(0) = X_7'(0) = X_7'(\infty) = 0\\
&Y_7(0) = Y_7(\infty) = 0
\end{aligned} \tag{VII}$$

Notice that equations (I) to (VII) are free of the parameters a_1 and a_2 and hence solutions to these equations are applicable for all values of a_1 and a_2.

Equations (I) to (VII) have been solved numerically on an IBM 7090 computer. It is noted that equations (I) are non-linear, while the remainder of the equations are linear (assuming that the solutions of equations (I), (II), and (III) are now known one after the other. Thus an integral equation approach was thought suitable for attempting the solution of equations (I), while initial value methods were indicated for the remaining equations.

If for ease in writing, one defines the quantities

$$B_1(\eta) = e^{-\int_0^\eta X_1(\eta)d\eta}$$

$$B_2(\eta) = [B_1(\eta)]^{Pr}$$

$$B_3(\eta) = \int_0^\eta \frac{\{[X_1'(\eta)]^2 - Y_1(\eta)\}}{B_1(\eta)} d\eta$$

$$K = \frac{\int_0^\infty B_3(\eta)B_1(\eta)d\eta}{\int_0^\infty B_1(\eta)d\eta},$$

a system of integral equations whose solution also satisfies (I) and which may be solved by a method of successive substitutions is

$$X_1(\eta) = \int_0^\eta X_1'(\eta)d\eta \tag{17a}$$

$$Y_1(\eta) = \frac{\int_\eta^\infty B_2(\eta)d\eta}{\int_0^\infty B_2(\eta)d\eta} \tag{17b}$$

$$X_1'(\eta) = \int_0^\eta [B_3(\eta) - K]B_1(\eta)d\eta. \tag{17c}$$

Guesses for the functional values $X_1'(\eta)$ were made. Values of X_1 and Y_1 were then obtained by (17a) and (17b) respectively. These values were then substituted in the right hand side of (17c) so as to obtain new values of $X_1'(\eta)$. (It was found that an under-relaxation factor of approximately 0.3 would insure convergence of the process when reasonable guesses for the values of $X_1'(\eta)$ had been given originally.)

After equations (I) had been solved fairly accurately by the method just outlined, one had in particular accurate approximations for $X_1''(0)$ and $Y_1'(0)$. Then even on the non-linear equations (I), initial value methods became very attractive. Actually, the Runge-Kutta method was used to obtain final results which are better than those obtained by the integral equation approach; they are better mainly because it was feasible to use a much smaller value of $\Delta\eta$ in the initial value method than in the integral equation method. Finally the Runge-Kutta method was used also to solve equations (II) through (VII).

For the problems in which we have been interested, the Prandtl number generally ranges from 0.7 to 1.0. In Tables I through VII, tabular results are given for the cases Pr = 0.7 and Pr = 1.0. In accordance with equations (16), linear combinations of the tabular results will be solutions of the original differential equations (12)-(14); the particular linear combination desired will depend on the values of a_1 and a_2 in the boundary conditions (15).

VELOCITY, TEMPERATURE AND HEAT TRANSFER

The velocity and temperature distribution in the boundary layer can be calculated by the equations

$$\frac{uR}{Gr^{\frac{1}{4}}\nu} = \bar{x}X_1' + \bar{x}^3[a_1X_2' + X_3'] + \bar{x}^5[a_2X_4' + a_1^2X_5' + a_1X_6' + X_7'] \tag{18}$$

$$\frac{T - T_\infty}{T_{wo} - T_\infty} = Y_1 + \bar{x}^2[a_1Y_2 + Y_3] + \bar{x}^4[a_2Y_4 + a_1^2Y_5 + a_1Y_6 + Y_7]. \tag{19}$$

The dimensionless heat transfer, the Nusselt number, can be computed from equation (20).

$$\frac{Nu}{Gr^{\frac{1}{4}}} = \frac{hR}{kGr^{\frac{1}{4}}} = -\{Y_1'(0) + \bar{x}^2[a_1Y_2'(0) + Y_3'(0)] + \bar{x}^4[a_2Y_4'(0) + a_1^2Y_5'(0) + a_1Y_6'(0) + Y_7'(0)\} \tag{20}$$

where

$$h = \frac{q}{T_{wo} - T_\infty}.$$

For Pr = 0.7,

$$\frac{Nu}{Gr^{\frac{1}{4}}} = .37023 + [.75688a_1 - .01609](\tfrac{x}{R})^2 + [.92847a_2 + .09471a_1^2 - .02885a_1 - .00009](\tfrac{x}{R})^4. \tag{21}$$

For Pr = 1.0,

$$\frac{Nu}{Gr^{\frac{1}{4}}} = .42143 + [.85200a_1 - .01861](\tfrac{x}{R})^2 + [1.0411a_2 + .10702a_1^2 - .03305a_1 - .00011](\tfrac{x}{R})^4. \tag{22}$$

DISCUSSION OF RESULTS

<u>Velocity and Temperature Profiles</u> The velocity and temperature profiles as computed from equations (18) and (19) for Pr = .7 at $\frac{x}{R} = 1$ are plotted in Figures 2 and 3. It is evident from Figures 2 and 3 that an increase in wall surface temperature will increase the maximum velocity in the boundary layer and will increase the absolute value of the temperature slope near the wall.

For $a_1 = a_2 = 0$ (isothermal case) and Pr = 0.7, a comparison of [2] and the present calculation reveals that the results listed in Tables I and III agree with those in [1] very well. However, there are significant discrepancies between the results of Table VII and those in [2]. When the initial values as given in [2] were used in the integration of equations (VII), it was found that the conditions at "free stream" were not satisfied. Consequently, it is believed that the values listed in Table III of [2] (corresponding to Table VII of the present paper) are in error. However, since the absolute values are quite small, this error may not have a significant effect on the overall results.

A comparison of velocity and temperature profiles in the boundary layer was made in [2] between the results of [1], [2], and the existing experimental data for $\bar{x} = \frac{\pi}{3}$ and $\frac{2\pi}{3}$. It was concluded that they agree very well with one another. This same conclusion is directly applicable to the present case of $a_1 = a_2 = 0$. As stated before, we know of no previous theoretical or experimental results for the case of a non-isothermal wall.

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<u>Heat Transfer</u> Figure 4 shows the effect of the Prandtl number on isothermal wall heat transfer. When the Prandtl number is increased from 0.7 to 1, the heat transfer is increased by about 13 to 14 percent, depending on the location. It is interesting to compare the effect of the Prandtl number on heat transfer from a vertical plate to that from a cylinder. It was derived by Eckert [7] and confirmed by Ostrach [3] that the effect of the Prandtl number on free convective heat transfer from an isothermal vertical plate may be approximately computed from

$$\left(\frac{Nu_x}{Gr_x}\right)\Big/\left(\frac{Nu_x}{Gr_x}\right)_{Pr=1} = \left(\frac{1.952}{.952 + Pr}\right)^{\frac{1}{4}} Pr^{\frac{1}{2}}. \tag{23}$$

If the Prandtl number is increased from 0.7 to 1, the heat transfer as computed from equation (23) would be increased by about 13 percent. In view of this close agreement on numerical results and in view of the fact that the heat transfer from a vertical plate and a horizontal cylinder are intimately related [7], it is recommended that in the absence of heat transfer results for the Prandtl numbers different from 0.7 and 1, equation (23) be used to compute the free convection heat transfer for different Prandtl numbers.

The effect of a non-isothermal wall on heat transfer may be computed from the following equation:

$$\left[\frac{q}{(T_w - T_\infty)Gr^{\frac{1}{4}}}\right]\Big/\left[\frac{q}{(T_w - T_\infty)Gr^{\frac{1}{4}}}\right]_{iso} = \left(\frac{T_{wo} - T_w}{T_w - T_\infty}\right)\frac{\left(\frac{Nu}{Gr^{\frac{1}{4}}}\right)}{\left(\frac{Nu}{Gr^{\frac{1}{4}}}\right)_{iso}} \tag{24}$$

where the subscript iso denotes a quantity corresponding to an isothermal wall $(a_1 = a_2 = 0)$. The quotient $(\frac{T_w - T_\infty}{T_{wo} - T_\infty})$ is prescribed, while the remaining quantities on the right hand side of equation (24) can be readily calculated by use of equations (21) and (22). The results of calculation for $Pr = 0.7$ and two combinations of a_1 and a_2 are shown in Figure 5. It is seen from Figure 5 that the wall temperature variation has a significant effect on heat transfer. For $a_1 = a_2 = 1$ at the location $\frac{x}{R} = 2$ (about 115°), the heat transfer is triple the value of that corresponding to the isothermal cylinder. Clearly, this indicates that the heat transfer in free convection from or to a non-isothermal cylinder may not be reliably calculated from solutions corresponding to an isothermal wall. This conclusion was drawn by Sparrow and Gregg [5] for the case of a vertical plate.

Comparison of Isothermal Heat Transfer As stated previously, the free convection heat transfer from an isothermal horizontal cylinder was first solved by Hermann [1]. It is therefore advisable to compare the present heat transfer results with those of [1]. Since Hermann's results are valid for $Pr = 0.74$, it is necessary to modify his results so that the comparison may be made based on the same Prandtl number (i.e., $Pr = 0.7$). Using equation (23), it is found that the correction factor is 0.976, i.e.,

$$\left(\frac{Nu}{Gr^{\frac{1}{4}}}\right)_{Pr=0.7} = 0.976\left(\frac{Nu}{Gr}\right)_{Pr=.74} \tag{25}$$

Using equation (25), the heat transfer results as given by Hermann and those of the present calculations for $Pr = 0.7$ are listed as follows:

① $\frac{x}{R}$		0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{11}{12}\pi$	π
$\frac{Nu}{Gr^{\frac{1}{4}}}$	② (Ref 1)	0.376	0.372	0.356	0.329	0.288	0.227	0.178	0
	③ Present	0.370	0.366	0.352	0.330	0.298	0.256	--	--
Ratio $\frac{②}{③}$		.984	.982	.992	1.004	1.035	1.127	--	--

It is seen from the above table that the heat transfer results agree with each other to within 3.5 percent for $\frac{x}{R} \leq \frac{2\pi}{3}$. Near $x = 0$, the present results are exact, and hence Hermann's calculations, after being corrected for the Prandtl number effect, underestimate the heat transfer by about 2 percent. For $\frac{x}{R} \geq \frac{2\pi}{3}$, Hermann's results are more accurate since the perturbation solution used in this paper breaks down for $\frac{x}{R}$ approaching π. In order to use the present solutions for heat transfer calculation at $\frac{x}{R} \geq \frac{2\pi}{3}$, it is suggested that the following formula be used:

$$\left(\frac{q}{T_w - T_\infty}\right)_{\bar{x} \geq 2\pi/3} = \gamma \left(\frac{q}{T_w - T_\infty}\right)_{\bar{x} = 2\pi/3} \quad . \qquad (26)$$

The correction factor γ can be derived from [1] and its values are as follows:

$\frac{x}{R}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{11}{12}\pi$	π
γ	1	.788	.620	0

The correction factor γ listed above is strictly for an isothermal cylinder. From physical considerations, the boundary layer is very thick near $x = \pi$ and hence heat transfer is very small even when the wall is non-isothermal. Therefore, before more reliable results are available, it is suggested that the above correction factor be also used for the non-isothermal case. Hence one can first compute the heat transfer for $\bar{x} = \frac{2\pi}{3}$ by use of equations (11) and (12) and then obtain the heat transfer for $\bar{x} > \frac{2\pi}{3}$ from equation (26) together with the correction factor γ given in the table.

In many practical problems the flow in the boundary layer far downstream of the stagnation point may be turbulent. In this case, the local heat transfer must be estimated from an appropriate turbulent heat transfer formula.

CONCLUSIONS

The boundary layer differential equations for free convective flows over a horizontal cylinder are solved by a perturbation method. The wall surface temperature is assumed to be varied in the manner of $a_1(\frac{x}{R})^2 + a_2(\frac{x}{R})^4$. A special transformation is used so that the solutions

are independent of the parameters a_1 and a_2, and hence results are applicable for all values of a_1 and a_2. It is found that the functional relationship between heat transfer and the Prandtl number for a cylinder is essentially the same as that for a vertical plate. Furthermore, the wall temperature variation has a significant effect on heat transfer and the heat transfer from a non-isothermal wall may not be computed from solutions corresponding to an isothermal wall.

ACKNOWLEDGMENT

The authors wish to express their appreciation to Mr. N. Carver for preparing the figures and tables in this paper.

NOMENCLATURE

a_1, a_2	Wall Surface Temperature Parameters, Eq. (1)
c_p	Specific Heat at Constant Pressure
F, G	Eqs. (11)
Gr	Grashof Number, $g\beta(T_{wo} - T_\infty)R^3/\nu^2$
g	Gravitational acceleration
h	Heat Transfer Coefficient, $h = q/(T_{wo} - T_\infty)$
k	Thermal Conductivity
M	Dimensionless Stream Function, Eq. (9)
Nu	Nusselt Number, $[q/(T_{wo} - T_\infty)](R/k)$
Pr	Prandtl Number, $\mu c_p/k$
q	Surface Heat Flux, $-(k\ \partial T/\partial x)_w$
R	Radius of Cylinder
T	Temperature
T_w	Surface Temperature
T_{wo}	Surface Temperature at Lower Stagnation Point
T_∞	Ambient Temperature
u, v	Velocity Components Along x and y Directions, Respectively
X, Y	Dependent Variables, Eq. (16)
x, y	Coordinates (Fig. 1)
$\bar{x}$	Dimensionless Coordinate, Eq. (9)
β	Expansion Coefficient
η	Dimensionless Coordinate, Eq. (9)
μ	Dynamic Viscosity

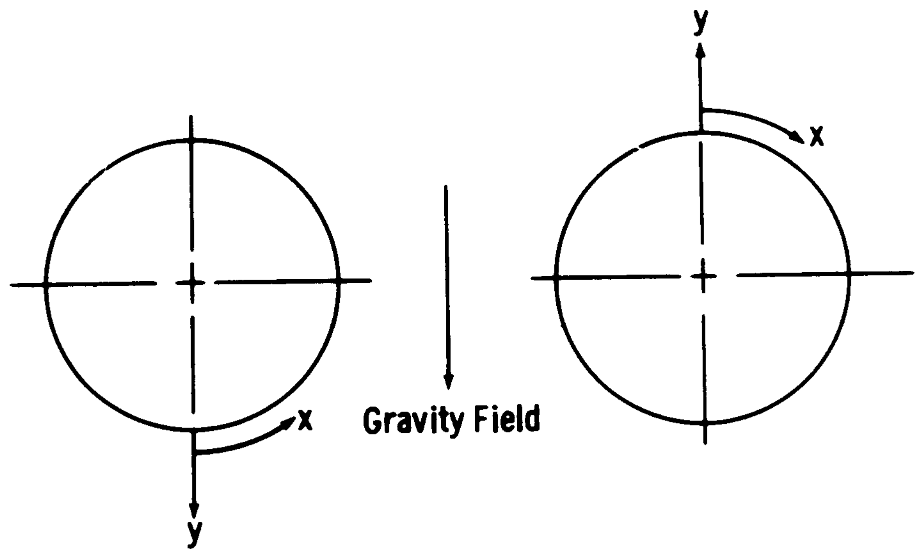
ν	Kinematic Viscosity, μ/ρ
ρ	Density
ϕ	Dimensionless Temperature, Eq. (9)
ψ	Stream Function, Eq. (6)

Subscripts

∞	at infinity
iso	for isothermal wall
o	at $x = 0$
w	at wall
wo	wall surface at $x = 0$

Superscript

'	differentiation with respect to η



(a) $T_W > T_\infty$ (b) $T_W < T_\infty$

Fig. 1. Coordinate Systems

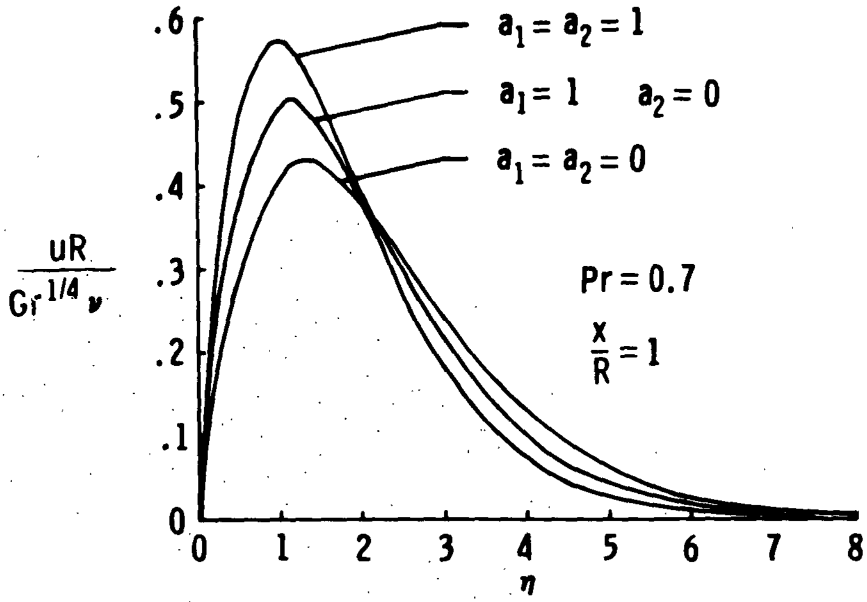


Fig 2. Velocity Profiles

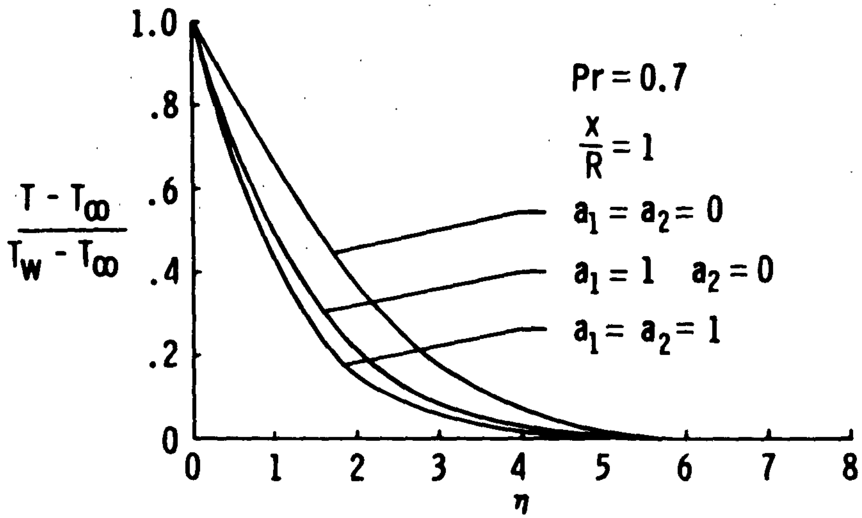


Fig 3. Temperature Profiles

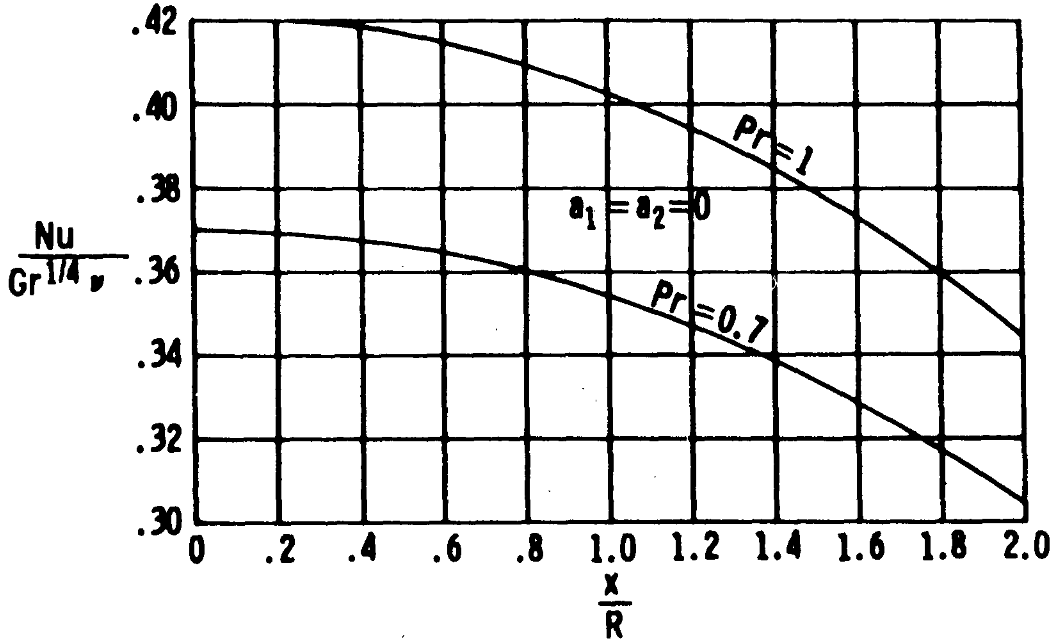


Fig 4. Effect of Prandtl Number on Heat Transfer

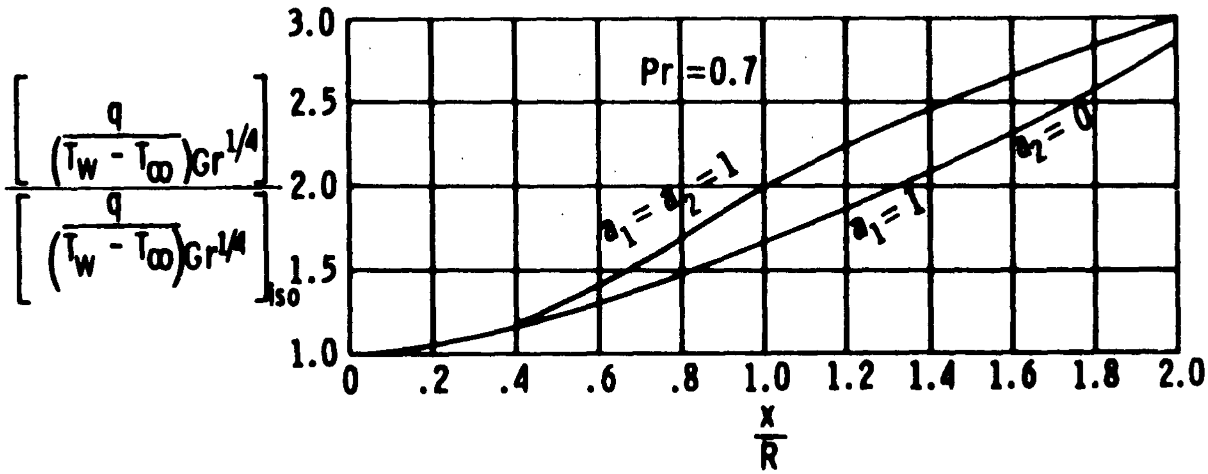


Fig 5. Effect of Nonisothermal Wall on Heat Transfer

TABLE I SOLUTIONS OF EQUATIONS (I)

η	Pr = 0.7					Pr = 1.0				
	X_1	X'_1	X''_1	Y_1	Y'_1	X_1	X'_1	X''_1	Y_1	Y'_1
0	0	0	.85934	1.0000	-.37023	0	0	.81701	1.0000	-.42143
.1	.00413	.08100	.76131	.96298	-.37020	.00392	.07677	.71922	.95786	-.42138
.2	.01588	.15241	.66762	.92597	-.36995	.01504	.14400	.62622	.91574	-.42100
.3	.03431	.21468	.57879	.88900	-.36932	.03242	.20219	.53846	.87368	-.42002
.4	.05853	.26834	.49517	.85212	-.36813	.05519	.25188	.45626	.83176	-.41820
.5	.08771	.31390	.41703	.81540	-.36626	.08253	.29364	.37982	.79007	-.41535
.6	.12106	.35193	.34452	.77890	-.36360	.11367	.32804	.30924	.74873	-.41131
.7	.15786	.38300	.27769	.74270	-.36008	.14791	.35568	.24456	.70785	-.40597
.8	.19744	.40766	.21653	.70691	-.35563	.18460	.37715	.18571	.66758	-.39928
.9	.23920	.42649	.16096	.67160	-.35024	.22316	.39301	.13258	.62804	-.39123
1.0	.28257	.44003	.11083	.63689	-.34391	.26304	.40385	.08501	.58938	-.38184
1.1	.32705	.44883	.06597	.60286	-.33665	.30378	.41019	.04278	.55172	-.37117
1.2	.37219	.45340	.02616	.56959	-.32851	.34495	.41257	.00563	.51518	-.35932
1.3	.41760	.45422	-.00886	.53718	-.31955	.38618	.41148	-.02670	.47989	-.34642
1.4	.46293	.45177	-.03937	.50571	-.30986	.42714	.40738	-.05452	.44593	-.33262
1.5	.50786	.44649	-.06564	.47523	-.29950	.46757	.40072	-.07814	.41339	-.31806
1.6	.55214	.43878	-.08798	.44582	-.28859	.50721	.39188	-.09788	.38233	-.30293
1.7	.59555	.42902	-.10668	.41753	-.27723	.54588	.38126	-.11407	.35282	-.28739
1.8	.63789	.41755	-.12207	.39039	-.26551	.58342	.36918	-.12703	.32486	-.27161
1.9	.67901	.40470	-.13442	.36444	-.25355	.61968	.35595	-.13707	.29850	-.25575
2.0	.71880	.39076	-.14404	.33968	-.24144	.65458	.34185	-.14450	.27371	-.23996
2.2	.79397	.36059	-.15619	.29382	-.21717	.71999	.31200	-.15270	.22882	-.20912
2.4	.86292	.32880	-.16062	.25278	-.19337	.77932	.28124	-.15380	.18995	-.17999
2.6	.92547	.29673	-.15920	.21641	-.17060	.83250	.25082	-.14968	.15667	-.15318
2.8	.98167	.26540	-.15352	.18445	-.14927	.87973	.22161	-.14194	.12850	-.12906
3.0	1.0317	.23552	-.14492	.15658	-.12964	.92128	.19420	-.13187	.10486	-.10778
3.2	1.0760	.20756	-.13448	.13247	-.11185	.95755	.16895	-.12050	.08520	-.08931
3.4	1.1149	.18180	-.12304	.11172	-.09594	.98901	.14604	-.10860	.06896	-.07351
3.6	1.1489	.15836	-.11126	.09397	-.08188	1.0161	.12551	-.09675	.05563	-.06015
3.8	1.1784	.13728	-.09960	.07885	-.06956	1.0394	.10731	.08534	.04475	-.04897
4.0	1.2039	.11849	-.08840	.06603	-.05888	1.0592	.09133	-.07463	.03592	-.03970
5.0	1.2865	.05394	-.04430	.02661	-.02453	1.1208	.03858	-.03471	.01166	-.01329
6.0	1.3230	.02315	-.02006	.01051	-.00982	1.1461	.01526	-.01453	.00370	-.00427
7.0	1.3384	.00958	-.00858	.00411	-.00387	1.1559	.00578	-.00571	.00116	-.00135
8.0	1.3447	.00388	-.00354	.00160	-.00151	1.1595	.00212	-.00215	.00036	-.00042
9.0	1.3473	.00154	-.00143	.00062	-.00059	1.1608	.00076	-.00079	.00011	-.00013
10.0	1.3483	.00061	-.00057	.00024	-.00023	1.1613	.00027	-.00028	.00004	-.00004
12.0	1.3488	.00009	-.00009	.00003	-.00003	1.1616	.00004	-.00003	.0	.0
14.0	1.3489	.00002	-.00001	.0	.0	1.1616	.00001	.0	.0	.0
16.0	1.3489	.0	.0	.0	.0	1.1616	.0	.0	.0	.0

TABLE II SOLUTIONS OF EQUATIONS (II)

η	Pr = 0.7					Pr = 1.0				
	X_2	X'_2	X''_2	Y_2	Y'_2	X_2	X'_2	X''_2	Y_2	Y'_2
0	0	0	.46848	1.0000	-.75688	0	0	.44346	1.0000	-.85200
.1	.00218	.04198	.37250	.92450	-.75125	.00205	.03949	.34793	.91506	-.84440
.2	.00809	.07479	.28532	.85008	-.73585	.00760	.06991	.26198	.83155	-.82380
.3	.01686	.09936	.20760	.77759	-.71272	.015766	.09222	.18612	.75063	-.79307
.4	.02772	.11664	.13959	.70773	-.68363	.02580	.10747	.12042	.67320	-.75477
.5	.03998	.12760	.08113	.64101	-.65013	.03706	.11664	.06461	.59885	-.71110
.6	.05306	.13317	.03183	.57780	-.61356	.04896	.12070	.01814	.53108	-.66390
.7	.06646	.13425	-.00889	.51836	-.57503	.06106	.12056	-.01968	.46714	-.61470
.8	.07979	.13166	-.04175	.46283	-.53550	.07296	.11703	-.04964	.40816	-.56476
.9	.09270	.12614	-.06750	.41127	-.49572	.08438	.11086	-.07258	.35418	-.51510
1.0	.10494	.11836	-.08695	.36367	-.45634	.09507	.10272	-.08937	.30511	-.46652
1.1	.11632	.10893	-.10089	.31997	-.41785	.10487	.09317	-.10084	.26081	-.41967
1.2	.12669	.09834	-.11009	.28006	-.38066	.11367	.08270	-.10778	.22110	-.37501
1.3	.13596	.08705	-.11526	.24379	-.34507	.12140	.07174	-.11095	.18572	-.33290
1.4	.14408	.07540	-.11708	.21099	-.31129	.12801	.06061	-.11102	.15442	-.29356
1.5	.15104	.06372	-.11616	.18146	-.27949	.13352	.04961	-.10860	.12692	-.25714
1.6	.15684	.05225	-.11304	.15502	-.24976	.13795	.03896	-.10425	.10290	-.22370
1.7	.16150	.04117	-.10821	.13144	-.22214	.14133	.02881	-.09844	.08208	-.19324
1.8	.16509	.03065	-.10210	.11052	-.19664	.14373	.01930	-.09158	.06415	-.16569
1.9	.16765	.02078	-.09507	.09204	-.17323	.14522	.01052	-.08404	.04884	-.14097
2.0	.16927	.01165	-.08744	.07581	-.15186	.14586	.00251	-.07609	.03587	-.11893
2.2	.16996	-.00424	-.07138	.04925	-.11493	.14495	-.01109	-.05993	.01590	-.08230
2.4	.16779	-.01691	-.05549	.02936	-.08508	.14164	-.02151	-.04453	.00237	-.05439
2.6	.16340	-.02651	-.04078	.01481	-.06141	.13654	-.02901	-.03074	-.00634	-.03373
2.8	.15737	-.03334	-.02782	.00445	-.04298	.13020	-.03394	-.01896	-.01151	-.01888
3.0	.15022	-.03777	-.01684	-.00268	-.02891	.12310	-.03674	-.00932	-.01419	-.00856
3.2	.14239	-.04021	-.00788	-.00735	-.01837	.11562	-.03781	-.00174	-.01516	-.00167
3.4	.13424	-.04105	-.00083	-.01021	-.01063	.10807	-.03756	.00398	-.01503	.00269
3.6	.12606	-.04066	.00452	-.01175	-.00508	.10066	-.03632	.00809	-.01421	.00525
3.8	.11804	-.03934	.00840	-.01235	-.00121	.09358	-.03441	.01086	-.01301	.00656
4.0	.11036	-.03738	.01105	-.01232	-.00139	.08693	-.03205	.01254	-.01164	.00702
5.0	.07944	-.02417	.01313	-.00841	.00465	.06149	-.01898	.01191	-.00548	.00470
6.0	.06117	-.01312	.00874	-.00446	.00309	.04767	-.00948	.00711	-.00220	.00210
7.0	.05170	-.00648	.00478	-.00215	.00163	.04108	-.00430	.00355	-.00083	.00083
8.0	.04715	-.00303	.00237	-.00099	.00078	.03818	-.00183	.00160	-.00030	.00031
9.0	.04506	-.00136	.00109	-.00044	.00036	.03696	-.00075	.00068	-.00011	.00011
10.0	.04413	-.00059	.00050	-.00019	.00016	.03647	-.00030	.00028	-.00004	.00004
12.0	.04356	-.00010	.00009	-.00004	.00003	.03621	-.00004	.00004	0	0
14.0	.04347	-.00002	.00002	0	0	.03616	-.00001	0	0	0
16.0	.04346	0	0	0	0	.03615	0	0	0	0

TABLE III SOLUTIONS OF EQUATIONS (III)

η	Pr = 0.7					Pr = 1.0				
	X_3	X'_3	X''_3	Y_3	Y'_3	X_3	X'_3	X''_3	Y_3	Y'_3
0	0	0	-.09340	0	.01609	0	0	-.08956	0	.01861
.1	-.00044	-.00852	-.07717	.00161	.01608	-.00042	-.00814	-.07338	.00186	.01860
.2	-.00165	-.01547	-.06198	.00322	.01604	-.00158	-.01471	-.05833	.00372	.01853
.3	-.00348	-.02096	-.04802	.00481	.01592	-.00332	-.01985	-.04457	.00556	.01835
.4	-.00580	-.02512	-.03540	.00640	.01570	-.00550	-.02367	-.03219	.00738	.01801
.5	-.00847	-.02808	-.02416	.00795	.01537	-.00801	-.02633	-.02124	.00916	.01750
.6	-.01138	-.03000	-.01431	.00947	.01491	-.01073	-.02797	-.01169	.01088	.01680
.7	-.01444	-.03099	-.00582	.01093	.01431	-.01358	-.02871	-.00352	.01251	.01590
.8	-.01755	-.03120	.00138	.01232	.01358	-.01645	-.02872	.00335	.01405	.01481
.9	-.02066	-.03076	.00736	.01364	.01272	-.01930	-.02809	.00900	.01547	.01354
1.0	-.02369	-.02977	.01222	.01486	.01175	-.02205	-.02695	.01353	.01675	.01211
1.1	-.02660	-.02835	.01605	.01599	.01068	-.02468	-.02542	.01703	.01789	.01057
1.2	-.02934	-.02659	.01897	.01700	.00952	-.02713	-.02358	.01963	.01886	.00892
1.3	-.03190	-.02458	.02108	.01789	.00830	-.02938	-.02152	.02143	.01967	.00723
1.4	-.03425	-.02240	.02247	.01865	.00703	-.03142	-.01931	.02252	.02031	.00551
1.5	-.03638	-.02010	.02325	.01929	.00575	-.03324	-.01703	.03202	.02077	.00381
1.6	-.03827	-.01776	.02351	.01980	.00447	-.03483	-.01473	.02300	.02107	.00215
1.7	-.03993	-.01542	.02333	.02019	.00320	-.03619	-.01244	.02258	.02121	.00057
1.8	-.04136	-.01311	.02279	.02045	.00198	-.03732	-.01022	.02181	.02119	-.00091
1.9	-.04256	-.01087	.02196	.02058	.00081	-.03824	-.00809	.02078	.02103	-.00227
2.0	-.04354	-.00872	.02090	.02061	-.00029	-.03894	-.00607	.01955	.02074	-.00350
2.2	-.04488	-.00479	.01834	.02035	-.00226	-.03979	-.00244	.01671	.01983	-.00552
2.4	-.04549	-.00141	.01546	.01973	-.00386	-.03996	.00060	.01366	.01857	-.00695
2.6	-.04548	.00139	.01253	.01883	-.00508	-.03959	.00303	.01065	.01708	-.00782
2.8	-.04497	.00361	.00973	.01772	-.00594	-.03879	.00487	.00787	.01547	-.00822
3.0	-.04407	.00530	.00719	.01648	-.00647	-.03767	.00620	.00541	.01382	-.00823
3.2	-.04289	.00651	.00496	.01516	-.00671	-.03634	.00706	.00331	.01220	-.00794
3.4	-.04150	.00730	.00306	.01381	-.00673	-.03487	.00754	.00159	.01066	-.00746
3.6	-.03999	.00775	.00151	.01248	-.00657	-.03334	.00772	.00022	.00922	-.00685
3.8	-.03842	.00793	.00026	.01119	-.00628	-.03180	.00766	-.00082	.00792	-.00617
4.0	-.03683	.00788	-.00070	.00997	-.00589	-.03029	.00741	-.00158	.00676	-.00548
5.0	-.02978	.00594	-.00253	.00521	-.00363	-.02399	.00502	-.00263	.00282	-.00260
6.0	-.02508	.00355	-.00208	.00251	-.00190	-.02018	.00273	-.00185	.00109	-.00107
7.0	-.02243	.00188	-.00127	.00116	-.00092	-.01823	.00132	-.00102	.00040	-.00041
8.0	-.02107	.00093	-.00068	.00052	-.00042	-.01732	.00059	-.00049	.00014	-.00015
9.0	-.02042	.00044	-.00034	.00023	-.00019	-.01692	.00025	-.00022	.00005	-.00005
10.0	-.02011	.00020	-.00016	.00010	.00008	-.01676	.00010	-.00009	.00002	-.00002
12.0	-.01992	.00004	.00003	.00002	.00002	-.01667	.00002	.00001	0	0
14.0	-.01989	0	0	0	0	-.01665	0	0	0	0
16.0	-.01988	0	0	0	0	-.01665	0	0	0	0

TABLE IV SOLUTIONS OF EQUATIONS (IV)

η	Pr = 0.7					Pr = 1.0				
	X_4	X'_4	X''_4	Y_4	Y'_4	X_4	X'_4	X''_4	Y_4	Y'_4
0	0	0	.40231	1.0000	-.92847	0	0	.38068	1.0000	-1.0411
.1	.00185	.03539	.30725	.90753	-.91744	.00174	.03325	.28615	.89639	-1.0263
.2	.00678	.06181	.22293	.81712	-.88810	.00635	.05762	.20326	.79555	-.98718
.3	.01395	.8036	.14998	.73035	-.84552	.01301	.07430	.13241	.69952	-.93119
.4	.02263	.09218	.08835	.64831	-.79392	.02100	.08449	.07334	.60968	-.86433
.5	.03220	.09839	.03752	.57175	-.73680	.02973	.08934	.02533	.52685	-.79144
.6	.04215	.10002	-.00334	.50105	.67693	.03872	.08990	-.01262	.45146	-.71624
.7	.05208	.09802	-.03524	.43638	-.61647	.04760	.08711	-.04166	.38359	-.64154
.8	.06166	.09323	-.05928	.37772	-.55704	.05606	.08182	-.06296	.32307	.56934
.9	.07066	.08639	-.07655	.32490	-.49981	.06390	.07474	-.07771	.26959	-.50102
1.0	.07889	.07811	-.08810	-.27766	-.44560	.07097	.06646	-.08699	.22271	-.43743
1.1	.08625	.06893	-.09491	.23566	-.39490	.07717	.05749	-.09181	.18193	-.37906
1.2	.09266	.05 926	-.09787	.19855	-.34802	.08246	.04822	-.09308	.14672	-.32608
1.3	.09810	.04946	-.09776	.16593	-.30506	.08682	.03897	-.09157	.11653	-.27847
1.4	.10256	.03979	-.09528	.13741	-.26601	.09026	.02997	-.08796	.09085	-.23606
1.5	.10607	.03046	-.09100	.11260	-.23077	.09282	.02142	-.08282	.06916	-.19857
1.6	.10867	.02163	-.08542	.09113	-.19915	.09456	.01344	-.07664	.05098	-.16568
1.7	.11041	.01340	-.07897	.07265	-.17095	.09554	.00612	-.06979	.03588	-.13701
1.8	.11137	.00585	-.07197	.05684	-.14593	.09581	-.00050	-.06259	.02345	-.11219
1.9	.11161	-.00098	-.06470	.04337	-.12384	.09546	-.00640	-.05530	.01333	-.09084
2.0	.11120	-.00708	-.05739	.03198	.10444	.09455	-.01156	-.04811	.00518	-.07260
2.2	.10873	-.01714	-.04327	.01441	-.07268	.09137	-.01981	-.03457	-.00632	-.04407
2.4	.10452	-.02448	-.03050	-.00238	-.04882	.08680	-.02550	-.02271	-.01302	-.02408
2.6	.09909	-.02945	-.01952	-.00553	-.03121	.08132	-.02902	-.01284	-.01639	-.01054
2.8	.09288	-.03242	-.01049	-.01043	-.01848	.07531	-.03078	-.00498	-.01755	-.00173
3.0	.08623	-.03377	-.00333	-.01317	-.00948	.06910	-.03114	.00101	-.01730	.00368
3.2	.07945	-.03387	.00213	-.01441	-.00329	.06292	-.03048	.00536	-.01623	.00673
3.4	.07249	-.03302	.00610	-.01463	.00082	.05695	-.02909	.00834	-.01472	.00817
3.6	.06629	-.03151	.00883	-.01418	.00341	.05132	-.02722	.01021	-.01303	.00858
3.8.	.06017	-.02956	.01056	-.01333	.00493	.04608	-.02506	.01121	-.01133	.00835
4.0	.05448	-.02734	.01150	-.01226	.00569	.04130	-.022278	.01156	-.00971	.00775
5.0	.03293	-.01605	.00994	-.00670	.00469	.02401	-.01231	.00862	-.00397	.00382
6.0	.02116	-.00817	.00589	-.00317	.00249	.01530	-.00578	.00464	-.00146	.00150
7.0	.01539	-.00385	.00300	-.00142	.00116	.01136	-.00250	.00217	-.00051	.00054
8.0	.01272	-.00173	.00142	-.00062	.00052	.00970	-.00103	.00093	-.00018	.00019
9.0	.01155	-.00075	.00064	-.00026	.00022	.00902	-.00041	.00038	.00006	.00006
10.0	.01104	-.00032	.00028	-.00011	.00010	.00875	-.00016	.00015	-.00002	.00002
12.0	.01074	-.00005	.00005	-.00002	.00002	.00860	.00003	.00002	0	0
14.0	.01070	0	.00001	0	0	.00857	.00001	0	0	0
16.0	.01070	0	0	0	0	.00855	0	0	0	0

TABLE V SOLUTIONS OF EQUATIONS (V)

η	Pr. = 0.7					Pr. = 1.0				
	X_5	X'_5	X''_5	Y_5	Y'_5	X_5	X'_5	X''_5	Y_5	Y'_5
0	0	0	-.04421	0	-.09471	0	0	-.04175	0	-.10702
.1	-.00022	-.00440	-.04368	-.00937	-.09177	-.00021	-.00416	-.04117	-.01057	-.10307
.2	-.00088	-.00870	-.04194	-.01820	-.08418	-.00083	-.00819	-.03930	-.02041	-.09302
.3	-.00195	-.01275	-.03896	-.02610	-.07354	-.00184	-.01197	-.03616	-.02904	-.07917
.4	-.00342	-.01645	-.03485	-.03285	-.06117	-.00321	-.01538	-.03190	-.03618	-.06332
.5	-.00523	-.01969	-.02983	-.03832	-.04808	-.00490	-.01833	-.02678	-.04169	-.04688
.6	-.00734	-.02232	-.02417	-.04247	-.03506	-.00686	-.02072	-.02109	-.04557	-.03086
.7	-.00968	-.02451	-.01814	-.04535	-.02267	-.00902	-.02254	-.01512	-.04790	-.01600
.8	-.01222	-.02602	-.01202	-.04704	-.01129	-.01134	-.02375	-.00912	-.04802	-.00275
.9	-.01487	-.02692	-.00602	-.04765	-.00119	-.01375	-.02437	-.00313	-.04851	-.00861
1.0	-.01758	-.02723	-.00034	-.04732	.00752	-.01620	-.02442	-.00206	-.04717	-.01800
1.1	-.02030	-.02700	-.00487	-.04619	-.01478	-.01862	-.02397	-.00692	-.04498	-.02541
1.2	-.02296	-.02628	-.00951	-.04441	-.02064	-.02098	-.02306	-.01116	-.04215	-.03095
1.3	-.02554	-.02512	-.01351	-.04211	.02515	-.02322	-.02176	-.01473	-.03885	-.03477
1.4	-.02798	-.02360	-.01683	-.03942	.02843	-.02532	-.02014	-.01760	-.03524	-.03708
1.5	-.03025	-.02178	-.01948	.03646	.03060	-.02724	-.01826	-.01978	-.03148	-.03806
1.6	-.03232	-.01972	-.02146	.03333	.03181	-.02896	-.01620	-.02132	-.02767	-.03795
1.7	-.03419	-.01750	-.02283	.03013	.03219	-.03048	-.01402	-.02226	-.02392	-.03695
1.8	-.03582	-.01518	-.02363	.02692	.03188	-.03177	-.01177	-.02265	-.02030	-.03526
1.9	.03722	-.01280	-.02392	.02377	.03102	-.03283	-.00950	-.02257	-.01688	-.03305
2.0	-.03838	-.01041	.02376	.02073	.02971	-.03367	.00727	-.02208	-.01370	-.03049
2.2	-.03999	-.00577	.02237	.01512	.02622	-.03469	-.00303	-.02016	-.00816	-.02485
2.4	-.04072	-.00153	.01996	.01028	.02211	-.03491	.00074	-.01741	-.00377	-.01915
2.6	-.04064	-.00217	.01696	.00628	.01792	-.03444	.00391	-.01426	-.00047	-.01395
2.8	-.03989	-.00524	.01374	.00310	.01397	-.03339	.00644	-.01104	-.00186	-.00950
3.0	-.03859	-.00767	.01056	.00067	.01046	-.03190	.00834	-.00798	-.00339	-.00591
3.2	-.03686	-.00948	.00760	.00112	.00749	-.03010	.00965	-.00524	-.00428	-.00316
3.4	-.03483	-.01073	.00497	.00236	.00505	-.02808	.01046	-.00289	-.00470	-.00114
3.6	-.03260	.01150	.00272	.00317	.00313	.02594	.01084	-.00096	-.00478	-.00026
3.8	-.03026	-.01185	.00086	.00365	.00166	.02376	.01087	-.00057	-.00463	-.00118
4.0	-.02788	.01187	-.00061	.00386	.00056	-.02161	.01063	-.00172	.00434	-.00172
5.0	-.01709	.00923	-.00367	.00310	.00137	-.01238	.00754	-.00363	.00237	-.00176
6.0	-.00968	.00568	-.00316	.00180	.00111	-.00655	.00428	-.00272	.00105	-.00091
7.0	-.00539	.00310	-.00200	.00093	.00064	-.00343	.00215	-.00157	.00043	-.00039
8.0	-.00312	.00158	-.00111	.00046	.00033	-.00191	.00100	-.00080	.00017	.00016
9.0	-.00200	.00076	-.00057	.00022	.00016	-.00122	.00044	-.00037	.00006	-.00006
10.0	-.00146	.00036	-.00028	.00010	.00008	-.00093	.00019	-.00016	.00002	-.00002
12.0	-.00110	.00007	-.00006	.00002	.00002	-.00075	.00003	-.00003	0	0
14.0	-.00103	.00001	-.00001	0	0	-.00073	0	0	0	0
16.0	.00103	0	0	0	0	.00073	0	0	0	0

TABLE VI SOLUTIONS OF EQUATIONS (VI)

	Pr. = 0.7					Pr. = 1.0				
η	X_6	X'_6	X''_6	Y_6	Y'_6	X_6	X'_6	X''_6	Y_6	Y'_6
0	0	0	-.05532	0	.02885	0	0	-.05255	0	.03305
.1	-.00025	-.00473	-.03951	.00286	.02825	-.00024	-.00445	-.03683	.00328	.03224
.2	-.00090	-.00797	-.02564	.00562	.02666	-.00084	-.00744	-.02320	.00640	.03009
.3	-.00180	-.00993	-.01391	.00817	.02434	-.00168	-.00917	-.01182	.00926	.02700
.4	-.00285	-.01082	-.00434	.01047	.02154	-.00264	-.00987	-.00266	.01178	.02330
.5	-.00394	-.01086	.00317	.01247	.01845	-.00363	-.00977	.00441	.01391	.01926
.6	-.00500	-.01025	.00878	.01416	.01522	-.00457	-.00906	.00959	.01563	.01511
.7	-.00597	-.00916	.01273	.01552	.01198	-.00542	-.00791	.01311	.01694	.01100
.8	-.00682	-.00775	.01525	.01655	.00883	-.00615	-.00648	.01524	.01784	.00708
.9	-.00751	-.00615	.01658	.01729	.00585	-.00672	-.00490	.01621	.01836	.00344
1.0	-.00805	-.00447	.01694	.01773	.00308	-.00712	-.00327	.01627	.01854	.00015
1.1	-.00841	-.00279	.01656	.01791	.00057	-.00737	-.00167	.01563	.01841	-.00275
1.2	-.00860	-.00117	.01562	.01786	-.00166	-.00746	-.00016	.01448	.01800	-.00523
1.3	-.00865	.00032	.01428	.01759	-.00360	-.00741	.00122	.01298	.01737	-.00730
1.4	-.00854	.00167	.01268	.01715	-.00525	-.00722	.00243	.01127	.01656	-.00895
1.5	-.00832	.00285	.01094	.01655	-.00661	-.00693	.00347	.00945	.01560	-.01021
1.6	-.00798	.00386	.00914	.01583	-.00770	-.00654	.00432	.00762	.01453	-.01110
1.7	-.00755	.00468	.00736	.01502	-.00853	-.00607	.00499	.00584	.01339	-.01165
1.8	-.00705	.00533	.00565	.01413	-.00913	-.00554	.00549	.00416	.01221	-.01191
1.9	-.00649	.00582	.00404	.01320	-.00951	-.00498	.00583	.00262	.01102	-.01190
2.0	-.00589	.00615	.00258	.01224	-.00971	-.00438	.00602	.00124	.00983	-.01169
2.2	-.00463	.00640	.00010	.01029	-.00962	-.00317	.00603	-.00102	.00758	-.01075
2.4	-.00336	.00623	-.00173	.00842	-.00907	-.00199	.00566	-.00261	.00556	-.00940
2.6	-.00216	.00575	-.00298	.00669	-.00821	-.00092	.00503	-.00359	.00383	-.00785
2.8	-.00107	.00507	-.00371	.00514	-.00719	.00001	.00426	-.00407	.00242	-.00630
3.0	-.00013	.00429	-.00404	.00381	-.00612	.00078	.00343	-.00417	.00130	-.00486
3.2	.00064	.00348	-.00406	.00269	-.00507	.00138	.00260	-.00400	.00046	-.00360
3.4	.00126	.00268	-.00386	.00178	-.00410	.00182	.00184	-.00365	-.00015	-.00254
3.6	.00172	.00194	-.00352	.00104	-.00324	.00212	.00115	-.00320	-.00057	-.00169
3.8	.00204	.00128	-.00311	.00047	-.00249	.00229	.00056	-.00271	-.00084	-.00103
4.0	.00224	.00070	-.00266	.00004	-.00186	.00235	.00007	-.00221	.00099	-.00053
5.0	.00197	-.00091	-.00073	-.00080	-.00015	.00167	-.00111	-.00036	-.00089	.00040
6.0	.00088	-.00113	.00014	-.00069	.00024	.00054	-.00106	.00030	-.00049	.00035
7.0	-.00013	-.00085	.00034	-.00044	.00023	-.00034	-.00070	.00036	-.00023	.00019
8.0	-.00081	-.00053	.00028	-.00025	.00015	-.00088	-.00039	.00025	-.00010	.00009
9.0	-.00122	-.00030	.00018	-.00013	.00009	-.00116	-.00020	.00014	-.00004	.00004
10.0	-.00144	-.00015	.00010	.00006	.00005	-.00130	-.00009	.00007	.00002	.00002
12.0	-.00160	-.00004	.00003	-.00001	.00001	-.00139	-.00002	.00002	0	0
14.0	-.00164	-.00001	.00001	0	0	.00140	0	0	0	0
16.0	.00164	0	0	0	0	.00140	0	0	0	0

TABLE VII SOLUTIONS OF EQUATIONS (VII)

η	Pr. = 0.7					Pr. = 1.0				
	X_7	X'_7	X''_7	Y_7	Y'_7	X_7	X'_7	X''_7	Y_7	Y'_7
0	0	0	.00246	0	.00007	0	0	.00236	0	.00011
.1	.00001	.00020	.00166	.00001	.00009	.00001	.00020	.00156	.00001	.00011
.2	.00004	.00034	.00095	.00002	.00009	.00004	.00032	.00086	.00002	.00012
.3	.00008	.00040	.00034	.00003	.00010	.00007	.00037	.00026	.00003	.00013
.4	.00012	.00041	-.00016	.00004	.00011	.00011	.00037	-.00022	.00005	.00014
.5	.00016	.00037	-.00054	.00005	.00012	.00014	.00033	-.00059	.00006	.00017
.6	.00019	.00030	-.00083	.00006	.00014	.00017	.00025	-.00086	.00008	.00020
.7	.00022	.00021	-.00102	.00008	.00017	.00019	.00016	-.00104	.00010	.00024
.8	.00023	.00010	-.00113	.00010	.00020	.00020	.00005	-.00114	.00013	.00029
.9	.00023	-.00002	-.00117	.00012	.00024	.00020	-.00006	-.00116	.00016	.00034
1.0	.00023	-.00013	-.00115	.00014	.00027	.00019	-.00018	-.00113	.00020	.00039
1.1	.00021	-.00024	-.00109	.00017	.00031	.00017	-.00029	-.00106	.00024	.00043
1.2	.00018	-.00035	-.00100	.00021	.00034	.00013	-.00039	-.00095	.00028	.00048
1.3	.00014	-.00044	-.00088	.00024	.00038	.00009	-.00048	-.00082	.00033	.00051
1.4	.00009	-.00052	-.00075	.00028	.00040	.00004	-.00056	-.00068	.00039	.00054
1.5	.00004	-.00059	-.00061	.00032	.00042	-.00002	-.00062	-.00053	.00044	.00055
1.6	-.00003	-.00064	-.00047	.00036	.00044	-.00008	-.00066	-.00038	.00050	.00055
1.7	-.00009	-.00068	-.00033	.00041	.00045	-.00015	-.00069	-.00024	.00055	.00054
1.8	-.00016	-.00071	-.00019	.00046	.00045	-.00022	-.00071	-.00010	.00060	.00052
1.9	-.00024	-.00072	-.00007	.00050	.00044	-.00029	-.00071	.00002	.00066	.00050
2.0	-.00031	-.00072	.00004	.00054	.00043	-.00036	-.00071	.00014	.00070	.00046
2.2	-.00045	-.00068	.00023	.00063	.00039	-.00050	-.00066	.00032	.00079	.00036
2.4	-.00058	-.00064	.00036	.00070	.00032	-.00063	-.00058	.00045	.00084	.00024
2.6	-.00070	-.00055	.00045	.00076	.00025	-.00073	-.00048	.00052	.00088	.00012
2.8	-.00081	-.00046	.00050	.00080	.00016	-.00082	-.00038	.00055	.00089	0
3.0	-.00089	-.00036	.00051	.00082	.00008	-.00088	-.00027	.00054	.00088	-.00010
3.2	-.00095	-.00026	.00049	.00083	.00001	-.00093	-.00016	.00050	.00085	-.00018
3.4	-.00099	-.00016	.00046	.00082	-.00006	-.00095	-.00007	.00045	.00081	-.00024
3.6	-.00101	-.00007	.00041	.00081	-.00012	-.00096	.00002	.00039	.00076	-.00028
3.8	-.00102	0	.00036	.00078	-.00016	-.00094	.00009	.00032	.00070	-.00031
4.0	-.00101	.00007	.00030	.00074	-.00020	-.00092	.00014	.00026	.00064	-.00032
5.0	-.00084	.00024	.00006	.00051	-.00024	-.00070	.00026	.00001	.00035	-.00024
6.0	-.00060	.00023	-.00005	.00030	-.00017	-.00045	.00022	-.00008	.00016	-.00013
7.0	-.00039	.00017	-.00007	.00017	-.00011	-.00027	.00014	-.00007	.00007	-.00006
8.0	-.00026	.00010	-.00006	.00008	-.00006	-.00016	.00008	-.00005	.00003	-.00003
9.0	-.00018	.00006	-.00004	.00004	-.00003	.00010	.00004	-.00003	.00001	-.00001
10.0	-.00013	.00003	-.00002	.00002	-.00002	-.00007	.00002	-.00001	0	0
12.0	-.00010	.00001	-.00001	0	0	-.00006	0	0	0	0
14.0	-.00009	0	0	0	0	-.00005	0	0	0	0
16.0	-.00009	0	0	0	0	-.00005	0	0	0	0

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